B.Sc. Semester-III Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 32113 **Course Code : SH/MTH/303/C-7**

Course Title : Numerical Methods

Time : 1 Hour 15 Minutes Full Marks : 25

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: $1\times5=5$

 a) State fourth order Runge-Kutta method for numerical solution of ordinary differential equation.

 b) Write down the approximate value of $\frac{5}{6}$ correct up to four significant figures and then find the percentage error in such approximation.

 c) Prove that $\Delta\nabla=\Delta-\nabla$, where the symbols have their usual meaning.

 d) State the condition of convergence of Gauss-Seidal iteration method for solving numerically a system of linear algebraic equations.

 e) Write down the iterative scheme of the *Fixed point iteration method* for finding a real root of an equation $f(x)=0$, stating the condition of convergence of the method.

 f) Give the geometrical interpretation of *Simpson*'s $\frac{1}{3}$rd rule of numerical integration.

 g) "Newton-Raphson method is said to have a quadratic convergence"– Explain why.

 h) Using Euler's method, find the value of y at x = 1.2, given that $\frac{dy}{dx}=y+2x$, $y(1)=1$, taking h = 0.2.

2. Answer any **two** questions: $5\times2=10$

 a) Using suitable Newton's Interpolation formulae, evaluate the values of $f(2.02)$ and $f(2.78)$ from the following tabular data: 5

x	2.0	2.2	2.4	2.6	2.8
$f(x)$	4.1003	4.1511	4.2027	4.2553	4.3091

 b) Solve by Gauss elimination method, the system

$$x+3y+2z=5$$
$$2x-y+z=-1$$
$$x+2y+3z=2$$

c) Establish the Newton-Cotes formula (closed type) in the form

$$I = (b-a)\sum_{i=0}^{n} k_i^{(n)} y_i$$

for the integral $I = \int_a^b f(x)\,dx$,

where $k_i^{(n)}$ are the Cotes coefficients and $y_i = f(x_i)$. Hence obtain the Trapezoidal formula for the given integral. 4+1

3. Answer any **one** question: 10×1=10

a) i) Give geometrical interpretation of Newton-Raphson method.

ii) Deduce the Lagrange's Interpolation formula. 4+6

b) i) Show that the n-th order divided difference for the function $y = f(x)$ for $(n+1)$ equi-spaced arguments $x = x_0, x_1, x_2, ..., x_n$ can be expressed as

$$f(x_0, x_1, x_2, ..., x_n) = \frac{\Delta^n y_0}{n!h^n}$$

where $x_i = x_0 + ih$ and $f(x_i) = y_i$ and h is the step length.

ii) Using *Euler's modified method*, find the value of $y(2.1)$ correct up to 4 decimal places from the following *initial value problem* (IVP) using h = 0.05

$$\frac{dy}{dx} = \frac{x-y}{x},\ y(2) = 2.$$ 4+6

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